Quark model study of the triton bound state

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Abstract

The three-nucleon bound state problem is studied employing nucleon-nucleon potentials derived from a basic quark-quark interaction. We analyze the effects of the nonlocalities generated by the quark model. The calculated triton binding energies indicate that quark-model nonlocalities can yield additional binding in the order of few hundred keV.

During the last decade the development of quark-model based interactions for the hadronic force has led to nucleon-nucleon (NN) potentials that provide a fairly reliable description of the on-shell data. As a consequence of the internal structure of the nucleon, such interaction models are characterized by the presence of nonlocalities. These nonlocalities are reflected in the off-shell properties and emerge from the underlying dynamics in a natural way.

The relevance and/or necessity of considering the nonlocal parts of nucleon-nucleon potentials in realistic interactions is still under debate. Indeed, over the past few years several studies have appeared in the literature which stress the potential importance of nonlocal effects for the quantitative understanding of few-body observables and, specifically, for the triton binding energy [1–5]. However, the majority of these investigations [1–4] explore only nonlocalities arising from the meson-exchange picture of the NN interaction. The effects of nonlocalities resulting from the quark substructure of the nucleon have only been addressed once so far [5] and more systematic studies are lacking altogether.

In this work we study the triton binding energy by means of a local and a nonlocal potential, derived from the same constituent quark-model. We will pay special attention to the nonlocal effects originating from the quark model. The nonlocal potential is derived by means of the resonating group method (RGM) and the local one is obtained through the Born-Oppenheimer approximation. These interactions are employed in Faddeev calculations of the three-nucleon binding energy.

I. QUARK-MODEL BASED NN POTENTIALS

The underlying idea of the quark model we use is that the constituent quark mass is a consequence of the spontaneous chiral symmetry breaking (SCSB). Then, between the chiral symmetry breaking scale ($\Lambda_{CSB} \sim 1$ GeV) and the confinement scale, ($\Lambda_C \sim 0.2$ GeV) QCD may be simulated in terms of an effective theory of constituent quarks interacting through the Goldstone modes associated with the SCSB. Perturbative features of QCD are incorporated through the one-gluon exchange potential. A more extensive description of the quark-model Hamiltonian can be found in the literature [7–9].

Based on the quark model Hamiltonian, two different procedures have been used in the literature to obtain baryonic interactions. The first one is the RGM. It allows to treat the inter-cluster dynamics in an exact way once the Hilbert space has been fixed. The potential derived from this method contains all the nonlocalities associated with quark antisymmetry. For the present study we will make use of the NN potential derived through a Lippmann-Schwinger formulation of the RGM equations in momentum space [9]. It reproduces the NN phase shifts up to 250 MeV lab energy with quite a good accuracy.

The second method is based on the Born-Oppenheimer approximation. It provides a clear-cut prescription for removing the nonlocalities while preserving the general properties of the interaction for lower partial waves, i.e., those coming form quark antisymmetry. This local interaction has been widely applied to a great variety of physical problems, obtaining reasonable results. In general, the phase shifts are reproduced with a comparable accuracy to the RGM results [6].

In both cases, for a correct description of the $^1S_0$ phase shift it is necessary to take into account the coupling to the $^5D_0$ $N\Delta$ channel [8], providing the required additional attraction. In order to achieve almost phase shift equivalence between the local and nonlocal interaction models, which is mandatory if one wants to reliably judge the influence of the nonlocalities, we have done a fine tuning of the potential parameters.

The results obtained for the two-body system with the local and nonlocal potentials are presented in Table I and Figs. 1 and 2. The $^1S_0$ and the $^3S_1 - {}^3D_1$ phase shifts and the low-energy scattering parameters as well as the deuteron binding energy are practically the same for both potential models, and also in very good agreement with experimental data.

II. TRITON BINDING ENERGY: RESULTS AND DISCUSSION

The three-body system is solved performing a five channel Fadeev calculation including the $^1S_0$ and $^3S_1 - {}^3D_1$ NN partial waves as input. Note that since in our model there is a coupling to the $N\Delta$ system, a fully consistent calculation would require the inclusion of two more three-body channels. However, their contribution to the $3N$ binding energy is known to be rather small [11] and therefore we neglect them for simplicity reasons.

To solve the three-body Faddeev equations in momentum space we first perform a separable finite-rank expansion of the $NN(-N\Delta)$ sector utilizing the EST method [12–14]. In Ref. [14] it was shown that with a separable expansion of sufficiently high rank, reliable and accurate results on the three-body level can be achieved. In the present case it turned out that separable representations of rank 6-8 for $^1S_0 - ({}^5D_0)$ and rank 6 for $^3S_1 - {}^3D_1$, are sufficient to get converged results. The results of the triton bound state calculation are summarized in Table II.

Let us first emphasize that the predicted binding energies for both models are comparable to those obtained from conventional potentials of the NN interaction such as the Paris, Bonn, or Nijmegen models [13,15]. Comparing the values for our local and nonlocal models, one observes that there is about 150 keV more binding for the nonlocal potential. Is this the enhancement we can expect from the nonlocalities due to the quark substructure of the nucleon? In order to answer this question we need to go back again to the NN results and scrutinize the on-shell properties carefully. For the ${}^1S_0$ partial wave the differences in the low-energy scattering parameters and in the phase shift are indeed very small, see Table I and Fig. 1 respectively.

Unfortunately, for the ${}^3S_1 - {}^3D_1$ partial wave the situation is much more complicated. While the deuteron binding energy and also the ${}^3S_1$ and ${}^3D_1$ phase shifts are in excellent agreement, (see Fig. 2) this cannot be said about the mixing parameter ε_1. In this case, it is difficult to estimate reliably the effect from the obvious deviation from phase equivalence on the triton binding energy.

However, one can clearly separate the effects from the two involved partial waves. For this purpose, we carried out additional $3N$ calculations where we combined the ${}^1S_0$ of the local model with the ${}^3S_1 - {}^3D_1$ of the nonlocal model and vice versa. Corresponding results are compiled in Table III where we show triton binding energies (in MeV) for different combinations of the local and nonlocal models. They strongly suggest that the nonlocalities present in the ${}^1S_0$ alone are already responsible for the enhancement of around 150 keV in the triton binding energy. The shift in the binding energy is independent of whether we use the local or nonlocal version model for the ${}^3S_1 - {}^3D_1$ partial wave. On the other hand, the nonlocalities present in the ${}^3S_1 - {}^3D_1$ partial wave seem to even decrease the binding energy. However, we suspect that here the effect of the nonlocalities is obscured by the fact that the two models are not strictly phase equivalent.

In summary, we have calculated the three-nucleon bound state problem utilizing NN potentials derived from a basic quark-quark interaction. One of these potentials was generated by means of the resonating group method so that nonlocalities resulting from the internal structure of the nucleon were preserved. The other potential is based on the Born-Oppenheimer approximation and is strictly local. These potentials are made nearly phase equivalent by fine tuning of some of the model parameters. The corresponding calculations of the triton bound state indicate that the nonlocalities resulting from the quark substructure of the nucleon yield additional attraction and, specifically, can lead to an increase of the binding energy by up to 200 keV. Thus, the effect of those nonlocalities on the three-nucleon binding energy is certainly appreciable. In particular, it is of the same magnitude as the one resulting from nonlocalities that occur in the meson-exchange picture of the NN interaction.

TABLES

TABLE I. NN properties

NN Low-energy scattering parameters				Deuteron properties		
		Local	Nonlocal		Local	Nonlocal
$^1S_0$	a_s (fm)	-23.758	-23.759	ϵ_d (MeV)	-2.2245	-2.2242
	r_s (fm)	2.694	2.682	P_D (%)	4.79	4.85
$^3S_1$	a_t (fm)	5.464	5.461	Q_d (fm^2)	0.280	0.276
	r_t (fm)	1.779	1.820	A_S (fm$^{-1/2}$)	0.900	0.891
				A_D/A_S	0.0243	0.0257

TABLE II. Properties of the three-nucleon bound state.

	E_B (MeV)	P_S (%)	$P_{S'}$ (%)	P_P (%)	P_D (%)
Local	-7.572	91.413	1.597	0.044	6.946
Nonlocal	-7.715	91.493	1.430	0.044	7.033

TABLE III.

		$^3S_1 - {}^3D_1$	
		Local	Nonlocal
$^1S_0$	Local	-7.572	-7.544
	Nonlocal	-7.745	-7.715

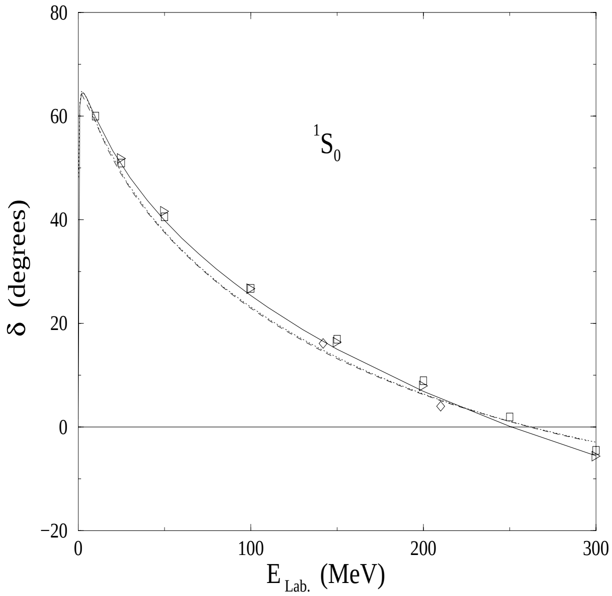

FIG. 1. NN phase shifts. The solid line stands for the nonlocal potential, the dashed line corresponds to the local one. The squares, diamonds and triangles are the experimental data taken from Refs. [10]. The dotted line shows the result of the EST separable representation of the local model.

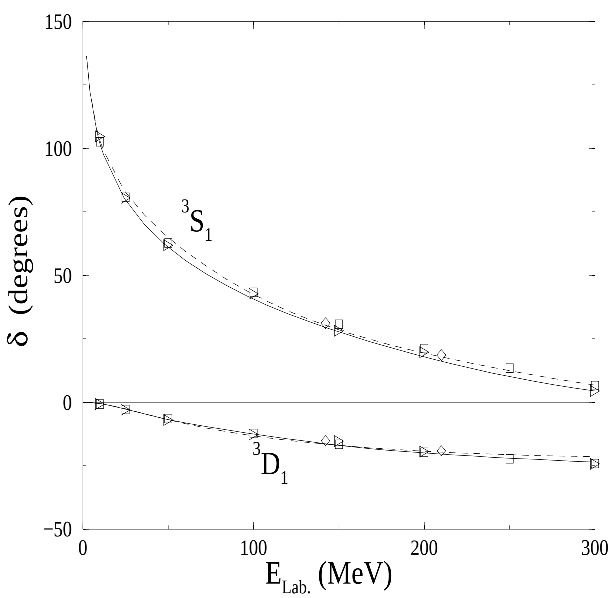

FIG. 2. NN phase shifts. The solid line stands for the nonlocal potential, the dashed line corresponds to the local one. The squares, diamonds and triangles are the experimental data taken from Refs. [10].

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